Keeper #5

Finding Inverses of Functions Given an Equation **Name:** ______________________ **Date:** ________

To find the inverse of a function given the equation, follow the 4 steps below.

Step 1: Replace $f(x)$ with y.

Step 2: Interchange x with y.
Swap

Step 3: Solve for y.

Step 4: If the inverse of f is a function, then replace y with $f^{-1}(x)$.

Step 5: Verify by checking to if $f(g(x)) = g(f(x))$

Example: $f(x) = 3x + 5$

$y = 3x + 5$

$x = 3y + 5$
$-5 \quad -5$

$x - 5 = 3y$
$\overline{3} \quad \overline{3}$

$\frac{x-5}{3} = y$

The parent function of the original equation is a linear function with a slope. So thinking of the graph of a linear function with any slope… it will pass the horizontal line test! Therefore the inverse is also a function. So we can replace y with $f^{-1}(x)$ to show that the inverse of $f(x)$ is also a function.

$\frac{x-5}{3} = f^{-1}(x)$ **or** $f^{-1}(x) = \frac{x-5}{3}$

Ex 1: $f(x) = 4x + 1$	Ex 2: $f(x) = -2x + 8$	Ex 3: $f(x) = \frac{1}{3}x + 7$
$y = 4x + 1$ $x = 4y + 1$ $-1 \quad -1$ $\frac{x-1}{4} = \frac{4y}{4}$ $\frac{x-1}{4} = y$ $f^{-1}(x) = \frac{x-1}{4}$	$y = -2x + 8$ $x = -2y + 8$ $-8 \quad -8$ $\frac{x-8}{-2} = \frac{-2y}{-2}$ $f^{-1}(x) = \frac{x-8}{-2} = y$	$y = \frac{1}{3}x + 7$ $x = \frac{1}{3}y + 7$ $-7 \quad -7$ $3(x-7) = 3(\frac{1}{3}y)$ $3x - 21 = y$ $f^{-1}(x) = 3x - 21$

a function is an expression where every input has exactly one output

function

x	y
-3	3
-2	2
-1	1

NOT A Function

x	y
-3	3
-3	2
-2	1
-2	0

function

x	y
-3	1
-2	1
-1	0

inverse

x	y
1	-3
1	-2
0	-1

Not a funct

Inverse Swaps x and y
Domain Range

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If the degree of the function is ODD the inverse is also a function.

Let's try a quadratic function. 

Ex: $f(x) = x^2 - 5$

$y = x^2 - 5$

$x = y^2 - \cancel{5}$

$+5 \qquad +\cancel{5}$

$x + 5 = y^2$

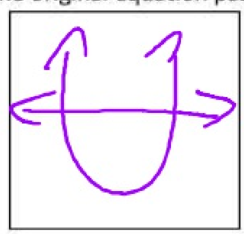
$\pm\sqrt{x+5} = \sqrt{y^{\cancel{2}}}$

$$\boxed{y = \pm\sqrt{x+5}}$$

Remember that the inverse operation of squaring a number is taking the $\pm\sqrt{}$ of that

When you decide if you should replace "y" with $f^{-1}(x)$ at the end, think… "Will the original equation pass the horizontal line test??" Draw a quick sketch of a quadratic function in the following box:

Does it pass the test???



Try these:

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Ex 1: $f(x) = x^2 + 1$	Ex 2: $f(x) = 2x^2 + 8$	Ex 3: $f(x) = 3x^2 + 5$
$y = x^2 + 1$ $x = y^2 + 1$ $-1 \quad -1$ $x - 1 = y^2$ $\pm\sqrt{x-1} = \sqrt{y^2}$ $y = \pm\sqrt{x-1}$	$y = 2x^2 + 8$ $x = 2y^2 + 8$ $-8 \quad -8$ $\frac{x-8}{2} \quad \frac{\cancel{2}y^2}{\cancel{2}}$ $\pm\sqrt{\frac{x-8}{2}} \quad \sqrt{y^2}$ $y = \pm\sqrt{\frac{x-8}{2}}$	$y = 3x^2 + 5$ $x = 3y^2 + 5$ $-5 \quad -5$ $\frac{x-5}{3} = \frac{3y^2}{3}$ $\pm\sqrt{\frac{x-5}{3}} = y$ $y = \pm\sqrt{\frac{x-5}{3}}$